Back Pressure

by Mike Prairie

Remember those "word problems" form high school math class? You know, "two trains are traveling on tracks, one eastward at 20 mph and one traveling north at 30 mph. If the tracks cross 6 miles from the north-bound train, how far in front of the east-bound train does the intersection need to be for the two trains to meet at the crossing at the same time?" (the answer is 4 miles, and the crash will happen in 12 minutes!)

The air flow in the sound hole is kind of like the trains. The acoustic flow moves in and out of the TSH, and the time for one cycle is 1/freq (1/370 = 0.0027 sec for F#4), and the jet from the flue flaps up and down once in the same period of 0.0027 sec. What happens when the jet reaches the cutting edge depends on what the acoustic flow was doing to the jet when it first emerged from the flue. In other words, the acoustic flow "disturbs" the jet flow at the flue, and the disturbance grows as it travels to the cutting edge at the average jet velocity of "u" inches/sec (use “u” for the velocity of the disturbance on the jet and “V” for the velocity of the air in the jet; they are related but different, which will be discussed below). If the disturbance in the right direction arrives at the cutting edge at the right time it will reinforce the oscillation of the air column. To get the timing right, we need to know how long it takes for the disturbance to travel from the flue to the cutting edge, which we can get by dividing the TSH length by the velocity.

A Disturbance on the Jet

The “disturbance” is an artifact of the jet entering a moving stream from the side. If you wanted to swim across a flowing river and get to a point directly across from where you started, you would have to angle the direction you swim so that you are going partly upstream instead of taking a straight path aimed at the spot directly across. To someone watching while floating by in a boat you would appear to be swimming up as well as across the river, but to someone watching on the bank, however, you would appear to be swimming straight across. The jet is similar in that the air in the jet appears to enter the TSH straight from the flue exit (before it slows down and starts to bend), but compared to the acoustic flow (seen by the observer on the “river”), the jet is actually moving partially upstream. Since the “river” is the acoustic flow that moves in and out of the TSH, its flow alternates in and out of the flute, alternating directions, speeding up and slowing down. The height of the disturbance does the same, changing the direction to the opposite of whatever the acoustic flow is doing, becoming its maximum when the acoustic flow is at its greatest.

So if a particle in the jet has a certain direction in the acoustic flow as it leaves the flue, the next particle will have a slightly different direction since the acoustic flow is constantly either speeding up or slowing down. The second particle pushes on the first as the first begins to slow down, but since they have slightly different levels of disturbance, they will have slightly different directions in the “river,” and the second particle will start to slide to one side of the first. When that happens, the second particle’s push will no longer be up the middle of the first, but instead it will be off center and will cause the first to begin to rotate and be deflected more strongly in the direction away from the second particle. This causes a “disturbance” that grows relatively quickly. If allowed to continue, the rotation will develop into vortices that will break away from the jet—preferably after reaching the cutting edge.

This is where it gets very complicated, and many researchers have spent significant parts of their careers trying to understand and model what is going on, beginning from before Lord Raleigh to modern studies using computational fluid dynamics (CFD). The book "The Physics of Musical Instruments" by Fletcher and Rossing summarizes the state of the knowledge at the time (1998) in a manner sufficient for us to get a pretty good feel for what happens in the flute, particularly for the velocity of the disturbance which is what we want to know at the moment. The equation that gives us a rough estimate is

$$u = \frac{V}{1 + \coth(kb)}$$

where u is the velocity of the disturbance on the jet, V is the velocity of the air in the jet, $k = 2\pi/\lambda$ is the wavenumber of the disturbance with λ being the wavelength of the disturbance, b is the half-height of the flue depth, and coth (kb) is a mathematical function called the "hyperbolic cotangent" of the "argument" kb. The wavelength of the jet disturbance is not the wavelength of the note being played—it is the length the jet would be if it had a full up-down cycle of the disturbance on it. Before using the formula, the wavelength of the disturbance needs to be found.

Speed, Length and Timing

In order for the flute to work properly, the arrival of the disturbance on the jet at the cutting edge must be at the same time as a particular part of the acoustic flow cycle. When the jet flips into the bore, it pushes on the air column, so it has to be pushing on the column as the acoustic flow is moving into the flute in order for the jet to reinforce the motion of the air column. If it flips down as the acoustic air flow is coming up, the jet will destroy the momentum in the air column. This is analogous to pushing a kid on a swing—you have to push at just the right time to make the swing go high. The same is true for the jet pushing on the air column.

Now if you blow too hard, the disturbance will reach the TSH too early, and if you blow too softly, it will arrive too late. In either case, the sound will be off the optimum, and if way too early or late it will support a different note (overblow), or none at all (just a soft hiss), respectively. So when is the optimum time to reach the cutting edge? If a downward disturbance on the jet that was created when the outward acoustical flow was at the maximum, that disturbance will do best to push on the air column when the inward acoustic air flow is at a maximum a half cycle later (0.0027/2 = 0.00135 sec for the F#). To achieve this if the TSH is 7/32 inches long, the average velocity of the disturbance should be (7/32 in) / 0.00135 sec = 162 in/sec.

For a nicely playing flute, this condition requires that the half-wavelength turns out to be slightly longer than the TSH length. So if the TSH is 7/32 inches long, the half-wavelength will be about 1/4 in, and that would make $\lambda = 1/2$ in. If the flue depth is 3/64 inches, we get $b = 0.0234$ in, so $kb = 0.295$ (kb is a dimensionless parameter in fluid dynamics called the "Strouhal number" that helps describe oscillating flows). Then $\coth(0.295) = 3.49$, so $u = 0.223V$. A plot of u as a fraction of V for a range of flue depths and three TSH lengths is shown below (keep in mind that the actual number is probably not accurate, but the trend seen by varying the parameters should be good).

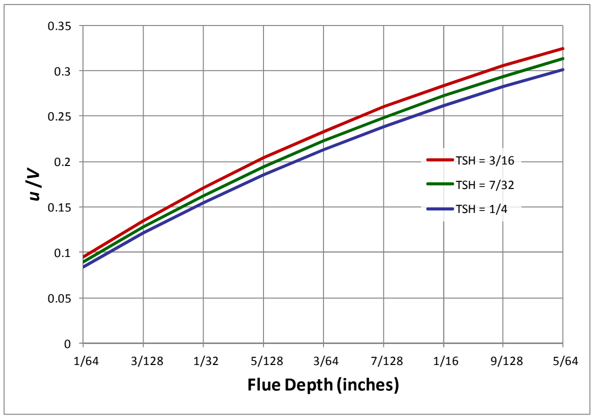

Figure 1. Disturbance velocity $\boldsymbol{u}$ as a fraction of $\boldsymbol{V}$ for various flue depths and TSH lengths.

Pressure

So our flute needs a disturbance velocity of about $u = 162\ in/sec = 0.223V$, then we find that $V = 726\ in/sec$, which is about 18.6 m/s in metric units. To relate that back to the pressure, Bernuolli's equation can be used, which for our purpose boils down to

$$p_{SAC} = \frac{1}{2}\rho V^2$$

where $\rho = 1.2\ kg/m^2$ is the density of air and p_{SAC} is the back pressure in the slow air chamber (SAC) in Pascals. In this case, $p_{SAC} = 208\ Pa$, which translates to 2.10 cm H_2O or 0.836 inches of H_2O. Repeating the chart from Figure 1 for backpressure results in the chart below.

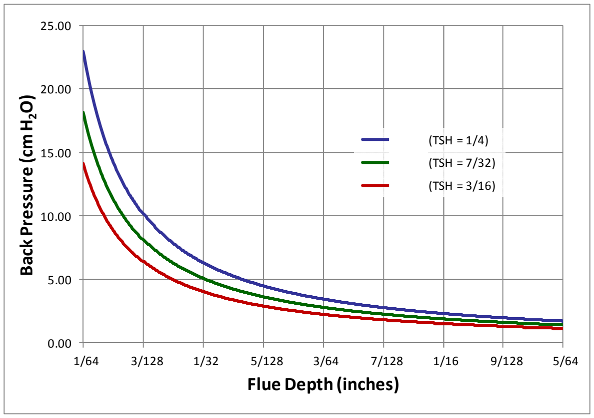

Figure 2. Back Pressure as a function of flue depth and TSH length for an F# flute.

To reiterate, the chart shows what is probably an accurate trend expected as the parameters are varied, but the absolute value of the backpressure is probably only a good estimate. Other things that will affect the back pressure are losses due to the drag on the walls of a long flue, the shape of the cutting edge that will affect the amount of bending the jet will experience, and other higher-order fluid dynamic effects (and probably a couple others). Also, this was for the fundamental note only. As higher-pitched notes are played, the disturbance needs to move faster, which means the initial jet velocity needs to be higher, which in turn will require slightly more pressure. This should also help explain the dynamics of breath control that we all learn to do subconsciously.